Economics Department

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1 Introduction[1]

1.1 In Search of Behavioural Foundations

The question of how people actually behave in the economy is still an open one. Experimental economics has clearly shown that economic agents do not obey the (traditional) strong rational requirements we used to impose (See R. Selten: Presidential address of the 97' European Economist Association Meeting). However, it is not necessarily preferable to assume that people are not rational. It might be better to assume that people are limited in their analytical capacity. This leads to the notion of *"bounded rationality"*. However, again it seems that people simply tend to escape a rigid definition of their behaviour. The definition of strong behavioural foundations appears to be, at least for the moment, impossible.

Though economic theory is not able to establish any foundation for the behaviour of the economic agents, it can provide useful criteria for classifying their different ways of behaving. Thus without going into the philosophical problem of the very motives for the (economic) behaviour of the agents, it is of interest to look at the type of behaviour that is likely to emerge in the economy.

Using an evolutionary "justification", the prevalent opinion is that the only behaviour that may "survive" in the economy is profit-maximizing behaviour. The equivalent of this term for the consumer being (individual) utility maximization. As already mentioned, significant departures from such behaviour are observed. These can be interpreted as being the result of several factors:

First, they might be due to limited information or bias *i.e.* to a limited

[1]This paper is a part of my Ph.D. Thesis. I would like to thank Louis Phlips for inviting me to present a previous version of this paper at his seminar. This chapter has also benefitted from helpful discussions at the 97' Meeting of the Southern European Economic Theorist Association (ASSET) and at the departmental seminar of the University of Evry (France). I am indebted to David Cass, John Cubbin, Benedetto Gui, Alan Kirman, Stephen Martin, Marco Scarsini, Spyros Vassilakis and Bruno Versaevel for their useful comments. All remaining errors are mine.

cognitive capacity of the agents. This is the notion of bounded rationality, the question being then whether such a factor is likely to disappear in the long term or whether these limitations might be overcome by a learning process. In terms of behaviour, the question is: what type of behavioural types might be learned in the economy?
Second, there is a question about the nature of the welfare of the economic agents. Some would consider it not unlikely that the agents are not indifferent to the situation of others, that managers behave to maintain their own standing with respect to others, etc. Such a question is of a more philosophical nature. Again, instead of taking a given position, it is of interest to know what type of behaviour is likely to emerge from interactions on the market.
The third factor that can be introduced to explain departures from complete rationality has to do with the definition of rationality itself. The situation in which all agents do their best to maximize their own profits is generally not the one that leads to the highest profits for all the (individual) agents. It might be true in a context where all economic agents were isolated. But whenever the agents effectively interact (even indirectly, by the mere fact of being part of the same economy) it is not the case. There is no reason why individual rationality should be able to coordinate to produce an outcome that is also rational for a group, because individual rationality looks exclusively at the consequences for itself. In the sense of S.-C. Kolm, this is already bounded rationality because agents do not look at *all* the consequences of their acts. The difference between individual rationality and group rationality is obvious when the prisoner's dilemma is considered. However we may consider the possibility of situations that involve both the "level" of rationality and their viability.

Instead of assuming here a given (fixed) behaviour, whatever justification one mights have for it, we look at how people will select their behaviour from a set of different possible strategies. People are actually assumed to be perfectly rational in the sense that they behave according their objectives and the payoffs they would *effectively* get according to the strategy they adopt. In that sense, there is no such things as con-

jectural variations. The agents find out what is their optimal behaviour within their economic environment. We do not assume information bias nor any limits to the cognitive capacity of the agents. We may postulate instead perfect and complete information. This assumption does not claim to be realistic, but it emphasises the fact that the results are not due to imperfect information or to a flawed learning process.
Results will be interpreted according to the three perspectives mentioned above: *capacity to discover* the structure of the economic environment, *emergence of a behavioural type* (cultural convention), and, propensity of the market to *coordinate* to produce *both individually-rational and group-rational* outcomes.

1.2 The setting

To make the analysis precise, this chapter studies the emergence of endogenous behavioural types in a rather specific framework, namely an oligopoly model *à la* Cournot. The reason why such a model was chosen is twofold: it is simple and it allows one to study interaction between agents in a well defined setting.
Several firms that produce a homogeneous good with an identical technology compete on the market. The production decisions are the result of a two-stage process. As no behaviour is assumed *a priori* for the firms, each economic agent selects one of the possible strategic profiles first and then decides on its production according to its (selected) behavioural type. Each firm attempts to maximize its profits and choose its behavioural type according to its profits in the existing economic environment. For the sake of simplicity, only two behavioural types compete. One is the classic Cournot-Nash behaviour where the firm strategically takes into account the effect of their own production on the price. The other implies that the firm behaves more "passively" and sets the production quantity by equating its marginal cost to prices. It corresponds to the price-taker behaviour in the general equilibrium model. An equilibrium is achieved when the distribution of strategic profiles is such that no firm has any interest to modify its behavioural type.

1.3 Interpretation and questions at hand

As already stated three interpretations can be given to the model, though they are clearly intrinsically connected.
The first perspective looks into the information problem. Though the computation of the Cournot strategy requires less information than in the joint-profit case, it still requires a knowledge of the slope of the demand curve in addition to individual firms' data. Accordingly, if this informational requisite is not claimed, the only justification for having firms that adopt a Cournot strategy is that this behavioural type happens to be more profitable for the firms that experiment it than an even simpler initial strategy. The strategy where firms set their quantities such that marginal costs equate with price would here serve as a basic initial strategy.
Note that the ability of firms to "learn" the Cournot equilibrium limits the domain of application of the general equilibrium model where it is assumed that firms rather than having a strategic behaviour, simply adopt a passive maximization assuming that prices are fixed. Hence this model provides some hints on the choice of market description.

The other perspective attempts to explain the emergence of managerial strategic types. Assume that firms are driven by managers. The manager's payoff results from a contract signed with the firms' owners, say once a year. The owners strive to maximize the firm's profits by choosing the proper incentives among the possible contracts. Only two types of contracts are proposed here. The first one gives the manager a payoff proportional to the profit, hence having no distortion with respect to the final objectives of the firm's owner. The other one gives a remuneration to the manager that takes account of both profits and sales, hence giving the firm a bias towards a more competitive behaviour than the standard Cournot-Nash.
It seems quite natural that even if there is no direct management and the firms' owners appoint managers to run the firms, it should be in their own interest to make them maximize what is their own objective *i.e.* the profits of the firm. Hence one could erroneously conclude that whether the firm's management is done by the owner of the firm or not has no

effects on the firm's strategy. In this view, whatever the structure of the firm, it is likely that firms end up reaching the Cournot-Nash equilibrium. However, the Cournot strategy rests on a conjecture that is wrong out of equilibrium, namely that the other firms have a fixed production level. Thus the Cournot-Nash strategy actually maximizes profits on condition that all the firms adopt the same strategy and produce at the Cournot-Nash level. If such a condition is not met, an alternative strategy might be more profitable.

The Cournot-Nash equilibrium is usually presented as the equilibrium concept that represents quantity-setting firms that "compete actively" in oligopoly. In the same way that the joint-profit equilibrium is subject to cheating because some more competitive strategies are more profitable to individual firms, the Cournot-Nash equilibrium is not preserved from individual "deviations". Thus if there is a possible commitment to some more competitive strategy, the latter is likely to emerge as a dominant strategy.

Technically, two questions arise. The first, that has already been presented, is the ability of firms to "learn" the Cournot Equilibrium *i.e.* the possibility of reaching the Cournot equilibrium, starting from a different strategic profile. The second is the stability of the Cournot equilibrium or the possibility that "active competition" brings firms to a less profitable equilibrium.

The third perspective looks at the coordination problem, as in R. Cooper's and A. John's 1988 paper. This problem arises at the level of the choice of strategic profiles. There is nevertheless a fundamental difference from the paper just mentioned. Given the strategic profiles, there is always *one* Nash equilibrium *only* in the game, in terms of quantity-setting. There are (possibly) several Nash equilibria depending on the strategic profiles that predominate in the economy. In other words, this (originally) convex problem loses this fundamental property because of the sequentiality of the decision process. As a result coordination difficulties arise in a setup that is expected to be structurally exempt from such problems.

The questions raised by these coordination failures are twofold. First, is

it possible for the agents in the economy to even consider the possibility of an alternative (perfectly viable) state of the economy? Second, how may people coordinate to shift from one equilibrium to the other? The robustness of an equilibrium (its stability) would then be assessed by the threshold number of people necessary to make the shift to an alternative strategy attractive.
Finally, there is a question about the possibility of calibrating structural models if the very fact of allowing a sequential decision process may multiply the number of possible equilibria.

1.4 Structure of the paper

The present note is organised as follows. In section 2, a model is offered that examines the choice of strategic types in an oligopoly. It is assumed that in the first period, firms commit themselves to a given type and in the second period compete on the market according to this strategic type. In this context, various results are established.
First, depending upon both the market structure and the distribution of "types", one "type" strictly dominates the other.
Second, Cournot equilibrium is only stable up to a threshold number of incumbents. This limits the domain of application of the Cournot model and defines when the competitive equilibrium model is to be used.
Third, situations may arise where *both* the Cournot Equilibrium and the equilibrium that comes up when all firms "cheat", are stable with respect to individual deviations of strategic type. This will cause a problem of equilibrium selection. Finally, while the stability of the Cournot-Nash equilibrium increases with the convexity of the cost-function, it appears that both equilibria are stable when the cost function is linear. However, if any firm decides its output by equating the marginal cost with the price, then all firms will find it profitable to follow in its footsteps.

In section 3, an application follows in which numerical examples are given. The possibility of hysteresis is shown, reinforcing the idea that, in some cases, historical parameters matter. A brief conclusion precedes a section that sums up the results in the form of tables and figures.

2 The model

2.1 A sequential decision process in a quantity setting oligopoly

Assume, as in a simple Cournot oligopoly model, that N firms produce an homogeneous good and face an inverse linear demand curve

$$P(Q) = A - BQ, \tag{1}$$

where Q is the total supply on the market. Firms are symmetric and use the same technology summarised for the sake of simplicity by a quadratic cost-function

$$C(q) = \alpha + \beta q + \gamma q^2, \tag{2}$$

where $\alpha,\ \beta,\ \gamma > 0$.

All firms are quantity setters that aim to maximise their profits given by the function

$$\pi(q, Q) = P(Q)q - C(q). \tag{3}$$

Assume two polar *strategic types*. The first one corresponds to the standard "Cournot strategy". Each firm maximizes its own profits, given that rivals hold their quantities fixed. This $\overline{\textit{type}}$ is labelled by a superscript. The other one corresponds to the "deviators"[2] and describes a more competitive strategy. Firms ignore completely the price elasticity and set quantities by equating marginal costs with price just as in the competitive equilibrium model. This $\underline{\textit{type}}$ is labelled by a subscript.
As explained above, the very fact that the Cournot strategy is not the optimal strategy in general provides a good justification for considering the possibility of alternative strategic *types*. Note that both strategic *types* that have been introduced here allow the firms to make positive

[2] In game theory literature, a "deviator" is a player who adopts a strategy that is more profitable for himself but that is less profitable if all players in the game were to adopt the same strategy. This explains why we attached such a name to the firms that adopt a $\underline{\textit{type}}$ strategic profile.

profits because the cost function is *not* assumed to be linear, but strictly convex.

The industry is characterized by a distribution $(\overline{n}, \underline{n})$ of strategic types among firms. It corresponds to the number $\overline{n}$ of firms that are "Cournot players" and the number $\underline{n}$ of "deviators" or "price takers"[3]. This distribution is assumed to be exogenously given as the result of some historical process unknown to us. A good argument for considering the possibility of having some "deviators" in the initial distribution is the fact that firms may not know the elasticity of the demand curve and therefore adopt in the first instance a more basic strategy, described here by *type*, that requires only the knowledge of price and of their own cost function. By allowing any distribution $(\overline{n}, \underline{n})$ at the initial stage of the game, various problems are contemplated; among others the stability of the Cournot-Nash equilibrium and the ability for firms to "learn" Cournot strategy starting from a more elementary strategy.
It is assumed that firms have *complete and perfect information* and *simultaneously and independently* consider changes in their *type* (*i.e.* firms play a best-reply *type* conditioned by the actual distribution).

In order to study the endogenous choice of types, the profits associated with each strategic type are now computed. Quantity decisions are obtained as a solution of the system of equations

$$P(Q) + \overline{q}\frac{\partial P(Q)}{\partial \overline{q}} - \frac{\partial C(\overline{q})}{\partial \overline{q}} = 0, \qquad (4)$$

$$P(Q) - \frac{\partial C(\underline{q})}{\partial \underline{q}} = 0, \qquad (5)$$

[3]Throughout this chapter "deviators" might be referred to as "price-takers" in reference to the general equilibrium model. *Type* corresponds indeed to a quantity setting strategy that is optimum if prices are assumed to be fixed. In the general equilibrium model, this is the assumption that is made. The usual justification for this is that if the number of players is "big enough" their effect on the price tends to be negligible. However such a strategy can be interpreted as due to a lack of information on the market demand or instead as a strategic (more profitable) alternative to the standard Cournot-Nash strategy as revealed by the model. This strategy cannot be eliminated merely because the number of players may be small.

where $Q = \overline{nq} + \underline{nq}$, (4) refers to $\overline{type}$ *i.e.* "Cournot players" and (5) refers to $\underline{type}$ *i.e.* "deviators" or "price-takers". Quantity setting of $\overline{type}$ corresponds to a manager payoff proportional to the profits of the firm. Quantity setting of $\underline{type}$ corresponds to a payoff proportional to $\pi + Bq^2$. For the type distribution $(\overline{n}, \underline{n})$, at equilibrium profits are

$$\overline{\pi}(\overline{n}, \underline{n}) = \frac{4\gamma^2 (\gamma + B)(A - \beta)^2}{\left[4\gamma^2 + 2\gamma(\underline{n} + \overline{n} + 1) B + \underline{n}B^2\right]^2} - \alpha \tag{6}$$

$$\underline{\pi}(\overline{n}, \underline{n}) = \frac{\gamma (2\gamma + B)^2 (A - \beta)^2}{\left[4\gamma^2 + 2\gamma(\underline{n} + \overline{n} + 1) B + \underline{n}B^2\right]^2} - \alpha \tag{7}$$

where $N = \overline{n} + \underline{n}$.

2.2 First results and comparison with some other models in industrial economics

In all instances, $\underline{\pi}(\overline{n}, \underline{n}) > \overline{\pi}(\overline{n}, \underline{n})$ holds. However, whether or not $\overline{\pi}(\overline{n}, \underline{n}) > \underline{\pi}(\overline{n} - 1, \underline{n} + 1)$ and $\underline{\pi}(\overline{n}, \underline{n}) > \overline{\pi}(\overline{n} + 1, \underline{n} - 1)$ hold depends on the distribution of types among the firms. As a result, the attractiveness of a move from one type to the other is endogenous.

This situation is similar to that described byC. d'Aspremont, A. Jacquemin, J. J. Gabszewicz and J.A. Weymark (1983). They propose a model where the competitive fringe reaps higher profits than members of the cartel but firms can find it profitable to join the cartel since the resulting increase in price would raise a cartel member's profit above what fringe firms currently receive. It is shown that with a finite number of firms there is always a stable dominant cartel. In the following the "Cournot players" play the role of cartel members, and "deviators" that of the competitive fringe. Note however that the analogy stops there because the results fundamentally differ.

The inequality $\underline{\pi}(\overline{n}, \underline{n}) > \overline{\pi}(\overline{n}, \underline{n})$ means that a deviator always makes more profit than a Cournot player. If $\overline{\pi}(\overline{n}, \underline{n}) > \underline{\pi}(\overline{n} - 1, \underline{n} + 1)$ holds, it means that it is not profitable for a Cournot player to "cheat" and adopt the deviators' strategic $\underline{type}$. If $\underline{\pi}(\overline{n}, \underline{n}) > \overline{\pi}(\overline{n} + 1, \underline{n} - 1)$ holds, it means that it is not profitable for a deviator to adopt the Cournot strategic $\overline{type}$. If both hold, neither Cournot players nor deviators have any interest in changing their strategic type. Hence the distribution of strategic types is in equilibrium. By contrast with the Cartel stability problem, it is shown that there is a threshold number of firms beyond which it is always more profitable for a firm to "cheat" rather than to adopt a Cournot strategic type. In other words, beyond a certain number of firms, the competitive equilibrium model is a correct representation of the market as $\underline{type}$ strictly dominates $\overline{type}$. This result is similar to that of R. Selten in his paper *"When four are few and six are many"*. In his article it is shown that a cartel might be stable up to four firms on the market whereas joint-cooperation becomes impossible when there are more than five firms. In this case firms are assumed to adopt the Cournot strategies. The differences with R. Selten are to be found in the diverse behavioural types under scrutiny and in the more general setup that evidences the dependence of the threshold number over structural parameters.

2.3 When behavioural types are endogenous...

To study the stability of the Cournot strategic type, let the incremental profit that accrues to a single firm by shifting from $\overline{type}$ to $\underline{type}$ be $\delta\pi_{\downarrow} \equiv \underline{\pi}\left(\overline{n}-1, \underline{n}+1\right) - \overline{\pi}\left(\overline{n}, \underline{n}\right)$. It is the incremental profit that accrues to a Cournot player firm that joins the group of deviators. We find

$$\delta\pi_{\downarrow} \geq 0 \quad \text{iff} \quad \underline{n} \geq f_N\left(\frac{\gamma}{B}\right) \tag{8}$$

where $f_N\left(\frac{\gamma}{B}\right) = \left(1+2\frac{\gamma}{B}\right)\sqrt{4\frac{\gamma}{B}\left(1+\frac{\gamma}{B}\right)} - 2\left(N-1\right)\frac{\gamma}{B}$ is the threshold number of $\underline{firms}$ beyond which all Cournot player firms find profitable to align on the corresponding $\underline{type}$ and join the deviators. For the incremental profit $\delta\pi_{\uparrow} \equiv \overline{\pi}\left(\overline{n}+1, \underline{n}-1\right) - \underline{\pi}\left(\overline{n}, \underline{n}\right)$ that accrues to a deviator who joins the group of Cournot players, the following result holds: $\delta\pi_{\uparrow} = -\left[\underline{\pi}\left(\overline{n}, \underline{n}\right) - \overline{\pi}\left(\overline{n}+1, \underline{n}-1\right)\right]$. Hence

$$\delta\pi_{\uparrow} \geq 0 \quad \text{iff} \quad \underline{n} \leq 1 + f_N\left(\frac{\gamma}{B}\right). \tag{9}$$

The two inequalities make it possible to derive various results.

For an equilibrium in the types' distribution to occur, the two conditions $\delta\pi_{\uparrow} \leq 0$ and $\delta\pi_{\downarrow} \leq 0$ have to be verified. This is clearly impossible. Hence, depending upon both the market structure and the distribution of types, one *type* strictly dominates the other. This result is in contrast with the cartel stability problem.

Three cases arise:

i. $(\overline{n}^*, \underline{n}^*) = (N, 0)$ iff $f_N\left(\frac{\gamma}{B}\right) \geq N-1$. i.e. $N \leq 1 + 2\sqrt{\frac{\gamma}{B}\left(1+\frac{\gamma}{B}\right)}$

ii. Either $(\overline{n}^*, \underline{n}^*) = (N, 0)$ or $(\overline{n}^*, \underline{n}^*) = (0, N)$ iff $1 \leq f_N\left(\frac{\gamma}{B}\right) \leq N$

iii. $(\overline{n}^*, \underline{n}^*) = (0, N)$ iff $f_N\left(\frac{\gamma}{B}\right) \leq 0$ i.e. $N > 1 + \left(1+\frac{B}{2\gamma}\right)\sqrt{4\frac{\gamma}{B}\left(1+\frac{\gamma}{B}\right)}$

In other words, in case (1), $\overline{type}$ is more profitable for all firms and "deviation" is never profitable. The Cournot-Nash equilibrium is the only stable equilibrium. Case (3) is symmetric: $\underline{type}$ is more profitable for all firms *i.e.* "deviation" is always profitable. The Cournot-Nash equilibrium is not stable as all firms prefer to behave as price-takers. In both cases there are no coordination problems. In the first case firms cannot fail to observe that the Cournot strategy is more profitable. This is due to the importance of demand elasticities. In the last case, even if all the firms perfectly well know that it would be more profitable for all firms to adopt $\overline{type}$, the Cournot equilibrium is not stable when faced with the (more competitive) $\underline{type}$.
In case (2), which type is adopted by all firms is *indeterminate*. Note that this indeterminacy is not due to imperfect or incomplete information but to coordination aspects. The profitability of shifts in types is a function both of structural parameters – such as brought in by the cost and demand specifications – *and* of the endogenous type distribution. Depending on the initial distribution of strategic types, the industry will end up either at the Cournot equilibrium or at the equilibrium where all firms adopt the deviators strategy. Both equilibria are stable against individual change of strategic type. In terms of information and learning,

it means that firms are not able to "learn" the (more profitable) Cournot behavioural $\overline{type}$ if they all enter the market with an insufficient knowledge for adopting a strategic profile different from $\underline{type}$. Note that firms are not aware of the sub-optimality of their strategies. In terms of behavioural types, this means that the strategic profile that appears to be more profitable is not necessarily the Cournot one. In terms of coordination, a "group-deviation" is necessary in order for the economy to shift from one equilibrium to the other. In any case, for both an "internal" or an hypothetical "external" observer, it is a rather arbitrary choice to decide which behavioural types correspond to the equilibrium values. It will depend on some exogenous factors that are not introduced in the model, such as the "history" of the industry. This question is addressed below by means of a numerical illustration.

Finally, note that case (1) becomes more likely when γ/B increases whereas case (2) is almost certain when γ/B goes to zero. When costs are linear the Cournot equilibrium is "metastable". No firms find it profitable to "deviate". However, if one does, all firms will shift to the more competitive strategic profile. This possibility of both behavioural types is what makes the difference from the work of P. D. Klemperer and M. A. Meyer (1986, 1989).

To sum up, the results are :

i. In contrast to the stability of cartels, *either* all firms will adopt Cournot strategies *or* all firms will prefer to "deviate" *i.e.* to behave as price-takers. In terms of incentives, this means that, for the managers, *only one type* of contract will prevail.
See Figures 3,4 and 5.

ii. Cournot equilibrium is only stable up to a threshold number of incumbents.
If the number of firms is high enough, all firms prefer to "cheat". This defines the domains of validity of both market representations: The Cournot model and the competitive equilibrium model.
See Figures 1,2 and 5.

iii. Both Cournot-Nash equilibrium and the equilibrium that arises when all firms prefer to deviate may be stable to an individual change of strategy with unchanged structural parameters. Hence there is a problem of equilibrium selection. The choice of the right strategy profile becomes a coordination game.
Note in particular that if the industry starts up with the basic strategy referred as $\underline{type}$, it is unlikely that it will "learn" the more sophisticated (and more profitable) $\overline{type}$.
Example: $\gamma/B = 0.8$ and $N = 4$. *See Figure 1 and 2.*

iv. The more convex the cost-function, the lower the incentive to "deviate" from Cournot Equilibrium.
The higher the number of firms, the higher the incentive to deviate. When costs are linear both equilibria are stable for any N and if any firm adopts $\underline{type}$, then all firms will find profitable to follow suit.
See Figure 5.

3 An example

Assume that $N = \overline{n} + \underline{n} = 5$ firms produce a homogeneous good and face an inverse linear demand-curve $P = A - BQ$, where $A = 150$ and $B = 2.5$. Firms are symmetric and use the same technology summarised by the quadratic cost-function $C(q) = 75 + 6q + 3q^2$. Individual profits for all possible distributions of types are given in Table 1. They were obtained from the expressions derived in (6) and (7) above.

Table 1: Cournot versus Price-Taker Strategy

Table 1	Number of firms adopting $\overline{type}$					
individual profits	$\overline{n} = 0$	$\overline{n} = 1$	$\overline{n} = 2$	$\overline{n} = 3$	$\overline{n} = 4$	$\overline{n} = 5$
$\overline{\pi}$		105.1	121.0	139.0	**159.7**	**183.6**
$\underline{\pi}$	**106.8**	**121.1**	**139.5**	159.3	182.0	

The decision in choosing the strategic type is made by comparing $\underline{\pi}(\overline{n})$ and $\overline{\pi}(\overline{n}+1)$. In order to facilitate the reading of the tables, the highest value is printed in bold characters.

Consider for example the distribution $(\overline{n}, \underline{n}) = (1, 4)$. Since $\underline{\pi}(0, 5) > \overline{\pi}(1, 4)$, individual rationality implies that the $\overline{firm}$ shifts to $\underline{type}$ to yield $(\overline{n}^*, \underline{n}^*) = (0, 5)$. Symmetrically, for $(\overline{n}, \underline{n}) = (4, 1)$, we have $\underline{\pi}(4, 1) < \overline{\pi}(5, 0)$. This in turn implies that the $\underline{firm}$ shifts to $\overline{type}$ to give $(\overline{n}^*, \underline{n}^*) = (5, 0)$. Both distributions $(5, 0)$ and $(0, 5)$ constitute stable equilibria.

In this example, both distributions $(5, 0)$ and $(0, 5)$ constitute stable equilibria. Which type is adopted by all firms may be described as resulting from a shock to demand that occurred in a previous period. This motivates the examination of the two following cases.

Case 1: Let $P = A - (B+\mathbf{1.5})\,Q$, *ceteris paribus*. Individual profits are given in Table 2.

Table 2: Negative Shock to Demand

Table 2	Number of firms adopting $\overline{type}$					
individual profits	$\overline{n}=0$	$\overline{n}=1$	$\overline{n}=2$	$\overline{n}=3$	$\overline{n}=4$	$\overline{n}=5$
$\overline{\pi}$		12.8	25.5	41.3	61.0	86.3
$\underline{\pi}$	**17.0**	**29.5**	**44.7**	**63.4**	**86.9**	

Since $\overline{\pi}\,(\overline{n},\underline{n}) < \underline{\pi}\,(\overline{n}-1,\underline{n}+1)$, we have $(\overline{n}^*,\underline{n}^*) = (0,5)$. All firms adopt $\underline{type}$.

A shock to the slope of the (inverse) demand curve may lead to $P = A - BQ$, that is to the set of individual profits described in Table 1. Since $\overline{\pi}\,(1,4) < \underline{\pi}\,(0,5)$, individual rationality implies that no $\underline{firm}$ shifts. Accordingly, $(\overline{n}^*,\underline{n}^*) = (0,5)$ is a stable equilibrium. Since $\overline{\pi}\,(5,0) < \underline{\pi}\,(4,1)$, $\overline{\pi}\,(4,1) < \underline{\pi}\,(3,2)$, $\overline{\pi}\,(3,2) < \underline{\pi}\,(2,3)$, $\overline{\pi}\,(2,3) < \underline{\pi}\,(1,4)$, and as already stated $\overline{\pi}\,(1,4) < \underline{\pi}\,(0,5)$, there is a "cascade" of deviations. All firms cheat, i.e. adopt $\underline{type}$ and $(\overline{n}^*,\underline{n}^*) = (0,5)$ is the only stable distribution of strategic type.

Case 2: Let $P = A - (B-\mathbf{1.5})\,Q$, *ceteris paribus*. Individual profits are given in Table 3.

Table 3: Positive Shock to Demand

Table 3	Number of firms adopting $\overline{type}$					
individual profits	$\overline{n}=0$	$\overline{n}=1$	$\overline{n}=2$	$\overline{n}=3$	$\overline{n}=4$	$\overline{n}=5$
$\overline{\pi}$		**442.0**	**455.8**	**470.3**	**485.33**	**501.0**
$\underline{\pi}$	439.1	452.7	466.9	481.6	497.0	

Since $\underline{\pi}\,(\overline{n},\underline{n}) < \overline{\pi}\,(\overline{n}+1,\underline{n}-1)$, we have $(\overline{n}^*,\underline{n}^*) = (5,0)$. All firms adopt $\overline{type}$.

Symmetrically, if a shock results in $P = A - BQ$, giving the set of individual profits described in Table 1, then $\underline{\pi}\,(4,1) < \overline{\pi}\,(5,0)$ implies that no $\overline{firm}$ shifts. Accordingly, $(\overline{n}^*,\underline{n}^*) = (5,0)$ is a stable equilibrium. In the same manner as above, it is possible to show that all firms will prefer $\overline{type}$. The distribution $(\overline{n}^*, \underline{n}^*) = (5,0)$ *i.e.* Cournot equilibrium is the only stable equilibrium.

The main result is that a distribution of strategic types should not be considered as determined by market structure parameters only. It was shown that two distinct distributions of types may arise in unchanged demand and technological conditions. Moreover, assume that the distribution of types is (say) $(\overline{n}, \underline{n}) = (0, N)$. Because $\overline{\pi}(N, 0) > \underline{\pi}(0, N)$ always holds, group rationality implies that all firms should adopt $\overline{type}$. When at the same time $\underline{\pi}(0, N) > \overline{\pi}(1, N-1)$, individual rationality implies that all firms remain stuck with $\underline{type}$. Furthermore, if $\underline{\pi}(1, N-1) > \overline{\pi}(2, N-2)$ a single move is not sufficient to align individual rationality with group rationality, that is to make a shift to $\overline{type}$ profitable for the other firms. Hysteresis arises unless a threshold value $\overline{n}_c \in [1, N-1]$ exists beyond which each firm in isolation finds it individually rational to shift. In other words if a distribution happens to be robust to individual moves, then coordination is necessary for all firms in the industry to shift to a more profitable behavioural type.

4 Conclusion

Such a simple model that offers only two strategic types does not pretend to reflect reality. However it helps to underline that Cournot-Nash equilibrium is to be considered as a convention rather than as the precise outcome of "active competition" in oligopolies. A frontier is drawn between the domain of application of the Cournot-model (or strategic models) and that of the competitive equilibrium model.

The possibility of having two stable equilibria in some cases suggests that coordination problems might arise in the market. Such a conjecture is to be considered with precaution as it may result from the *discrete* set of strategic types presented. However it seriously questions the ability of market competition as a selection mechanism and thus as a factor of welfare, however it might be defined. Of importance is the fact that the problem under scrutiny was structurally convex. This property is lost because the decision process is assumed to be sequential. Such a result suggests that coordination problems might be more common than one might think at first sight.

5 References

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6 Tables and Figures

Table **B1**:	$P = 150 - 2.5Q$	$C(q) = 6q + 0.03q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 2$	$\underline{n} = 0$	**917.91**	-
	$\underline{n} = 1$	4.20	90.72
	$\underline{n} = 2$	-	**24.30**
$N = 3$	$\underline{n} = 0$	**518.38**	-
	$\underline{n} = 1$	4.02	86.79
	$\underline{n} = 2$	1.10	**23.74**
	$\underline{n} = 3$	-	**10.88**
$N = 4$	$\underline{n} = 0$	**332.56**	-
	$\underline{n} = 1$	3.85	83.12
	$\underline{n} = 2$	1.08	**23.21**
	$\underline{n} = 3$	0.50	**10.72**
	$\underline{n} = 4$	-	**6.15**
$N = 5$	$\underline{n} = 0$	**231.31**	-
	$\underline{n} = 1$	3.69	79.67
	$\underline{n} = 2$	1.05	**22.69**
	$\underline{n} = 3$	0.49	**10.55**
	$\underline{n} = 4$	0.28	**6.08**
	$\underline{n} = 5$	-	**3.94**
$N = 6$	$\underline{n} = 0$	**170.14**	-
	$\underline{n} = 1$	3.54	76.43
	$\underline{n} = 2$	1.03	**22.19**
	$\underline{n} = 3$	0.48	**10.40**
	$\underline{n} = 4$	0.28	**6.01**
	$\underline{n} = 5$	0.18	**3.91**
	$\underline{n} = 6$	-	**2.74**
$N = 7$	$\underline{n} = 0$	**130.37**	-
	$\underline{n} = 1$	3.40	73.38
	$\underline{n} = 1$	1.01	**21.71**
	$\underline{n} = 1$	0.47	**10.24**
	$\underline{n} = 1$	0.28	**5.94**
	$\underline{n} = 1$	0.18	**3.87**
	$\underline{n} = 1$	0.13	**2.72**
	$\underline{n} = 7$	-	**2.02**

Table B1: $P = 150 - 2.5Q$	$C(q) = 6q + 0.03q^2$		
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 8$	$\underline{n} = 0$	**103.08**	-
	$\underline{n} = 1$	3.27	70.52
	$\underline{n} = 2$	0.98	**21.24**
	$\underline{n} = 3$	0.47	**10.09**
	$\underline{n} = 4$	0.27	**5.87**
	$\underline{n} = 5$	0.18	**3.84**
	$\underline{n} = 6$	0.13	**2.70**
	$\underline{n} = 7$	0.09	**2.00**
	$\underline{n} = 8$	-	**1.55**
$N = 9$	$\underline{n} = 0$	**83.53**	-
	$\underline{n} = 1$	3.14	67.81
	$\underline{n} = 2$	0.96	**20.79**
	$\underline{n} = 3$	0.46	**9.94**
	$\underline{n} = 4$	0.27	**5.80**
	$\underline{n} = 5$	0.18	**3.80**
	$\underline{n} = 6$	0.12	**2.68**
	$\underline{n} = 7$	0.09	**1.99**
	$\underline{n} = 8$	0.07	**1.54**
	$\underline{n} = 9$	-	**1.22**
$N = 10$	$\underline{n} = 0$	**69.07**	-
	$\underline{n} = 1$	3.02	65.26
	$\underline{n} = 2$	0.94	**20.35**
	$\underline{n} = 3$	0.45	**9.79**
	$\underline{n} = 4$	0.27	**5.74**
	$\underline{n} = 5$	0.17	**3.77**
	$\underline{n} = 6$	0.12	**2.66**
	$\underline{n} = 7$	0.09	**1.98**
	$\underline{n} = 8$	0.07	**1.53**
	$\underline{n} = 9$	0.06	**1.22**
	$\underline{n} = 10$	-	**0.99**

Table 4: Strategic Equilibria and Payoffs for Quasi-Linear Cost Functions

Table **B2**:	$P = 150 - 2.5Q$	$C(q) = 7.5 + 6q + 0.3q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 2$	$\underline{n} = 0$	**877.44**	-
	$\underline{n} = 1$	161.84	476.83
	$\underline{n} = 2$	-	**190.87**
$N = 3$	$\underline{n} = 0$	**509.24**	-
	$\underline{n} = 1$	123.95	368.46
	$\underline{n} = 2$	51.26	**160.57**
	$\underline{n} = 3$	-	**87.31**
$N = 4$	$\underline{n} = 0$	**330.83**	-
	$\underline{n} = 1$	97.49	292.77
	$\underline{n} = 2$	42.92	**136.72**
	$\underline{n} = 3$	22.02	**76.93**
	$\underline{n} = 4$	-	**47.86**
$N = 5$	$\underline{n} = 0$	231.08	-
	$\underline{n} = 1$	78.28	**237.84**
	$\underline{n} = 2$	36.24	**117.60**
	$\underline{n} = 3$	18.95	**68.16**
	$\underline{n} = 4$	10.20	**43.14**
	$\underline{n} = 5$	-	**28.75**
$N = 6$	$\underline{n} = 0$	169.73	-
	$\underline{n} = 1$	63.90	**196.71**
	$\underline{n} = 2$	30.80	**102.05**
	$\underline{n} = 3$	16.34	**60.69**
	$\underline{n} = 4$	8.75	**38.99**
	$\underline{n} = 5$	4.29	**26.21**
	$\underline{n} = 6$	-	**18.06**
$N = 7$	$\underline{n} = 0$	129.32	-
	$\underline{n} = 1$	52.85	**165.11**
	$\underline{n} = 2$	26.32	**89.23**
	$\underline{n} = 3$	14.10	**54.27**
	$\underline{n} = 4$	7.48	**35.33**
	$\underline{n} = 5$	3.49	**23.93**
	$\underline{n} = 6$	0.91	**16.55**
	$\underline{n} = 7$	-	**11.49**

Table B2:	$P = 150 - 2.5Q$	$C(q) = 7.5 + 6q + 0.3q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 8$	$\underline{n} = 0$	101.31	-
	$\underline{n} = 1$	44.18	**140.32**
	$\underline{n} = 2$	22.58	**78.54**
	$\underline{n} = 3$	12.16	**48.72**
	$\underline{n} = 4$	6.34	**32.09**
	$\underline{n} = 5$	2.77	**21.88**
	$\underline{n} = 6$	0.42	**15.16**
	$\underline{n} = 7$	−1.20	**10.51**
	$\underline{n} = 8$	-	**7.16**
$N = 9$	$\underline{n} = 0$	81.09	-
	$\underline{n} = 1$	37.26	**120.51**
	$\underline{n} = 2$	19.43	**69.52**
	$\underline{n} = 3$	10.46	**43.88**
	$\underline{n} = 4$	5.33	**29.20**
	$\underline{n} = 5$	2.12	**20.02**
	$\underline{n} = 6$	−0.02	**13.90**
	$\underline{n} = 7$	−1.52	**9.61**
	$\underline{n} = 8$	−2.61	**6.49**
	$\underline{n} = 9$	-	**4.16**
$N = 10$	$\underline{n} = 0$	66.03	-
	$\underline{n} = 1$	31.64	**104.44**
	$\underline{n} = 2$	16.75	**61.85**
	$\underline{n} = 3$	8.98	**39.64**
	$\underline{n} = 4$	4.43	**26.62**
	$\underline{n} = 5$	1.53	**18.33**
	$\underline{n} = 6$	−0.43	**12.73**
	$\underline{n} = 7$	−1.81	**8.77**
	$\underline{n} = 8$	−2.82	**5.87**
	$\underline{n} = 9$	−3.59	**3.68**
	$\underline{n} = 10$	-	**1.99**

Table 5: Strategic Equilibria and Payoffs for "low" Decreasing-Returns-to-Scale

Table **B3**:	$P = 150 - 2.5Q$	$C(q) = 75 + 6q + 3q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 2$	$\underline{n} = 0$	**550.78**	-
	$\underline{n} = 1$	**464.34**	515.41
	$\underline{n} = 2$	-	439.12
$N = 3$	$\underline{n} = 0$	**370.50**	-
	$\underline{n} = 1$	**317.70**	354.89
	$\underline{n} = 2$	**273.76**	306.79
	$\underline{n} = 3$	-	266.33
$N = 4$	$\underline{n} = 0$	**258.23**	-
	$\underline{n} = 1$	**223.65**	251.93
	$\underline{n} = 2$	**194.19**	219.68
	$\underline{n} = 3$	**168.88**	191.97
	$\underline{n} = 4$	-	168.00
$N = 5$	$\underline{n} = 0$	**183.61**	-
	$\underline{n} = 1$	**159.75**	181.98
	$\underline{n} = 2$	139.06	159.31
	$\underline{n} = 3$	120.95	**139.51**
	$\underline{n} = 4$	105.07	**122.12**
	$\underline{n} = 5$	-	**106.76**
$N = 6$	$\underline{n} = 0$	131.52	-
	$\underline{n} = 1$	114.36	**132.29**
	$\underline{n} = 2$	99.25	**115.75**
	$\underline{n} = 3$	85.88	**101.12**
	$\underline{n} = 4$	74.00	**88.11**
	$\underline{n} = 5$	63.38	**76.48**
	$\underline{n} = 6$	-	**66.06**
$N = 7$	$\underline{n} = 0$	97.71	-
	$\underline{n} = 1$	80.96	**95.73**
	$\underline{n} = 2$	69.61	**83.30**
	$\underline{n} = 3$	59.45	**72.18**
	$\underline{n} = 4$	50.32	**62.19**
	$\underline{n} = 5$	42.10	**53.19**
	$\underline{n} = 6$	34.65	**45.04**
	$\underline{n} = 7$	-	**37.64**

Table B3:	$P = 150 - 2.5Q$	$C(q) = 75 + 6q + 3q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 8$	$\underline{n} = 0$	65.41	-
	$\underline{n} = 1$	55.68	**68.06**
	$\underline{n} = 2$	46.93	**58.48**
	$\underline{n} = 3$	39.03	**49.83**
	$\underline{n} = 4$	31.88	**42.00**
	$\underline{n} = 5$	25.37	**34.88**
	$\underline{n} = 6$	19.44	**28.39**
	$\underline{n} = 7$	14.03	**22.46**
	$\underline{n} = 8$	-	**17.03**
$N = 9$	$\underline{n} = 0$	43.68	-
	$\underline{n} = 1$	36.09	**46.60**
	$\underline{n} = 2$	29.20	**39.07**
	$\underline{n} = 3$	22.94	**32.21**
	$\underline{n} = 4$	17.22	**25.95**
	$\underline{n} = 5$	11.99	**20.23**
	$\underline{n} = 6$	7.19	**14.98**
	$\underline{n} = 7$	2.78	**10.15**
	$\underline{n} = 8$	−1.28	**5.70**
	$\underline{n} = 9$	-	**1.59**
$N = 10$	$\underline{n} = 0$	26.62	-
	$\underline{n} = 1$	20.59	**29.64**
	$\underline{n} = 2$	15.07	**23.60**
	$\underline{n} = 3$	10.02	**18.07**
	$\underline{n} = 4$	5.38	**13.00**
	$\underline{n} = 5$	1.11	**8.32**
	$\underline{n} = 6$	−2.82	**4.01**
	$\underline{n} = 7$	−6.46	**0.03**
	$\underline{n} = 8$	−9.83	**−3.66**
	$\underline{n} = 9$	−12.96	**−7.08**
	$\underline{n} = 10$	-	**−10.27**

Table 6: Strategic Equilibria and Payoffs for "medium" Decreasing-Returns-to-Scale

Table B4:	$P = 150 - 2.5Q$	$C(q) = 25 + 9q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 2$	$\underline{n} = 0$	**372.92**	-
	$\underline{n} = 1$	**362.30**	368.15
	$\underline{n} = 2$	-	357.80
$N = 3$	$\underline{n} = 0$	**305.04**	-
	$\underline{n} = 1$	**297.00**	301.86
	$\underline{n} = 2$	**289.26**	294.00
	$\underline{n} = 3$	-	286.42
$N = 4$	$\underline{n} = 0$	**253.15**	-
	$\underline{n} = 1$	**246.92**	251.03
	$\underline{n} = 2$	**240.90**	244.92
	$\underline{n} = 3$	**235.08**	239.01
	$\underline{n} = 4$	-	233.29
$N = 5$	$\underline{n} = 0$	**212.60**	-
	$\underline{n} = 1$	**207.68**	211.19
	$\underline{n} = 2$	**207.91**	206.35
	$\underline{n} = 3$	**198.28**	201.65
	$\underline{n} = 4$	**193.80**	197.10
	$\underline{n} = 5$	-	192.68
$N = 6$	$\underline{n} = 0$	**180.32**	-
	$\underline{n} = 1$	**176.36**	179.40
	$\underline{n} = 2$	**172.51**	175.50
	$\underline{n} = 3$	**168.78**	171.70
	$\underline{n} = 4$	**165.15**	168.02
	$\underline{n} = 5$	**161.62**	164.43
	$\underline{n} = 6$	-	160.95
$N = 7$	$\underline{n} = 0$	**154.19**	-
	$\underline{n} = 1$	**150.96**	153.62
	$\underline{n} = 2$	**147.82**	150.42
	$\underline{n} = 3$	**144.75**	147.32
	$\underline{n} = 4$	**141.78**	144.29
	$\underline{n} = 5$	**138.87**	141.35
	$\underline{n} = 6$	**136.05**	138.48
	$\underline{n} = 7$	-	135.68

Table B4:	$P = 150 - 2.5Q$	$C(q) = 25 + 9q^2$	
Number of Firms	Number of 'deviators'	Payoff of a Cournot player	Payoff of a 'deviator'
$N = 8$	$\underline{n} = 0$	**132.75**	-
	$\underline{n} = 1$	**130.08**	132.42
	$\underline{n} = 2$	**127.48**	129.78
	$\underline{n} = 3$	**124.94**	127.20
	$\underline{n} = 4$	**122.46**	124.69
	$\underline{n} = 5$	**120.05**	122.24
	$\underline{n} = 6$	**117.69**	119.85
	$\underline{n} = 7$	**115.39**	117.51
	$\underline{n} = 8$	-	115.24
$N = 9$	$\underline{n} = 0$	**114.94**	-
	$\underline{n} = 1$	**112.71**	114.79
	$\underline{n} = 2$	**110.53**	112.57
	$\underline{n} = 3$	**108.40**	110.41
	$\underline{n} = 4$	**106.32**	108.30
	$\underline{n} = 5$	**104.29**	106.24
	$\underline{n} = 6$	**102.31**	104.23
	$\underline{n} = 7$	**100.37**	102.26
	$\underline{n} = 8$	**98.47**	100.34
	$\underline{n} = 9$	-	98.46
$N = 10$	$\underline{n} = 0$	**99.98**	-
	$\underline{n} = 1$	**98.10**	99.96
	$\underline{n} = 2$	96.26	98.09
	$\underline{n} = 3$	94.45	96.26
	$\underline{n} = 4$	92.69	**94.47**
	$\underline{n} = 5$	90.97	**92.72**
	$\underline{n} = 6$	89.28	**91.01**
	$\underline{n} = 7$	87.63	**89.33**
	$\underline{n} = 8$	86.02	**87.69**
	$\underline{n} = 9$	84.44	**86.09**
	$\underline{n} = 10$	-	**84.52**

Table 7: Strategic Equilibria and Payoffs for "High" Decreasing-Returns-to-Scale

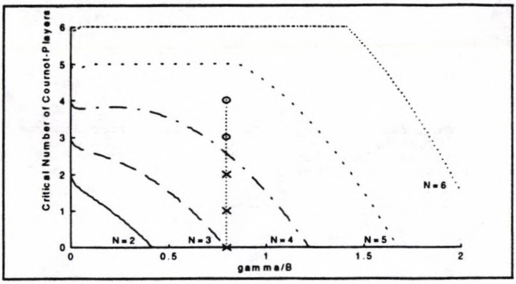

Figure 1: Minimum Number of Cournot Players to Ensure the Stability of Cournot Equilibrium

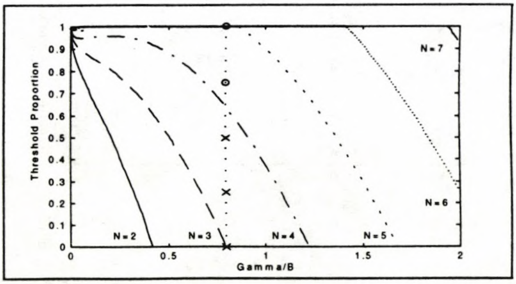

Figure 2: Threshold Proportion of Cournot Players to Ensure the Stability of Cournot Equilibrium

$\overline{type}$ is dominant when the number of firms that adopt this strategic $\overline{type}$ is higher than the minimum given by the curve relating to the corresponding total number of firms in the industry N. (Plot of the point $(\gamma/B, \overline{n})$ is to the right and above the curve). Consider for example the particular value $\frac{\gamma}{B} = 0.8$. If $N = 2$ or $N = 3$, all firms adopt $\overline{type}$. If $N = 4$, all firms find it profitable to shift to $\overline{type}$ when $\underline{n} \leq 2$ and all firms adopt $\underline{type}$ when $\underline{n} > 2$. If $N \geq 5$ all firms adopt $\underline{type}$.

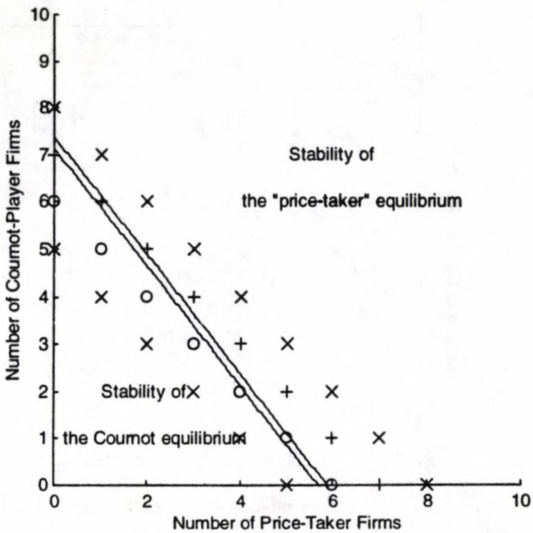

Figure 3: Map of the Stability Regions ($\gamma/B = 2$)

In both regions of the plan $(\overline{n}, \underline{n})$ firms find it profitable to align on the corresponding type (i.e. $\overline{type}$ where Cournot equilibrium is stable, $\underline{type}$ in the other). At the frontier between both regions, there is high instability because it is profitable for both groups to shift to the other strategic type.

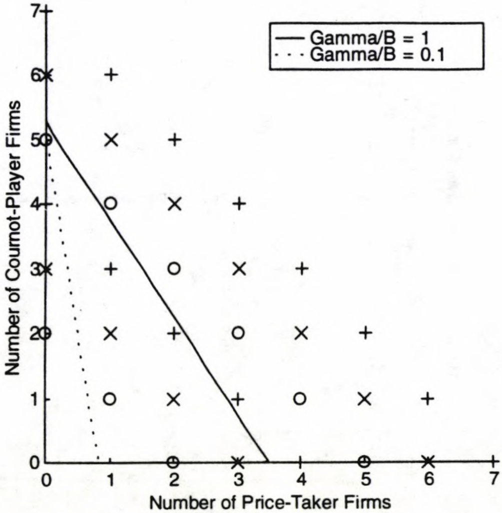

Figure 4: Metastability of the Cournot Equilibrium for Quasi-Linear Cost Functions

In both regions of the plan $(\overline{n}, \underline{n})$ firms find it profitable to align on the corresponding type (i.e. $\overline{type}$ where Cournot equilibrium is stable, $\underline{type}$ in the other). When the frontier passes through the lines $\overline{n} + \underline{n} = N$, it means that for the number of firms, both equilibria are possible. This is especially likely when the cost functions are only slightly convex. (See the frontier for $\gamma/B = 0.1$).

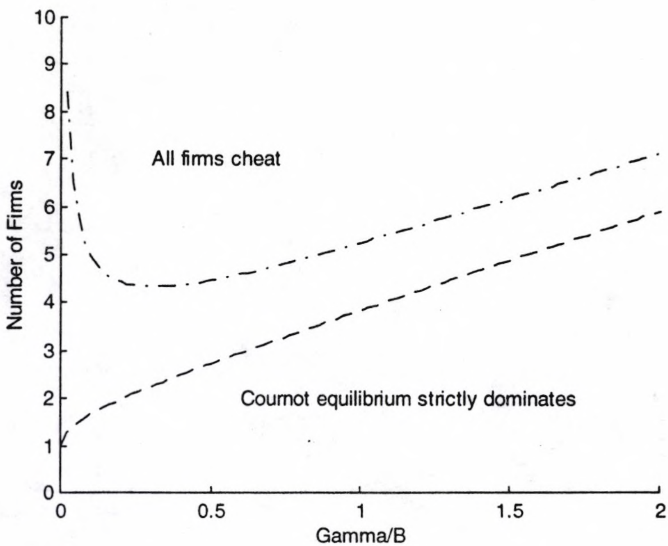

Figure 5: Number of Firms, Convexity of the Cost Functions and Stability of the Cournot-Nash Equilibrium.

In both regions firms find it profitable to align on the corresponding type (i.e. $\overline{type}$ where Cournot equilibrium is stable, $\underline{type}$ in the other). In between the two lines both equilibria are possible, which is always the case when the cost function is linear. For any strictly convex technology, there is a threshold number of firms that makes the Cournot equilibrium unstable.

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